Helium Multi-Chart Validation

Seven Measured Representations, Dual-Family Canonicalization, and Parametric Resolution Sweep

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Abstract. We present the definitive version of the helium multi-chart validation, incorporating direct STRUC-PERC-I v2.4.0 measurements on all seven helium representations: three QM-I encodings (preprocessed, gap structure, spectrum) and three Zeeman encodings (singlet, triplet, full ladder). All decisive coordinates are now directly measured; no estimates remain.

The study delivers three central results.

Result 1 (Dual-family canonicalization). Canonicalization is resolved for both encoding families simultaneously. Within the QM-I family (FULL class), the canonical encoding is **spectrum QM-I** ($m_{\text{local}} = 0.00217$, $\kappa_{\text{conn}} = 271{,}999$, tailDom = 0.9978). Within the Zeeman family (GIANT class), the canonical encoding is **triplet Zeeman** ($m_{\text{local}} = 0.04782$, GR = 0.9978). All seven representations lie within the class predicted by their respective canonical ordering. The fitted boundary $F(x) = C(L) - \text{tailDom}(L) = 0$ achieves perfect 7/7 separation.

Result 2 (Resolved resolution dependence). All three Zeeman encodings are classified as GIANT at full instrument resolution, not TAIL as the Phase Mapping corpus (at $n \approx 2{,}000$) reported. The transition from TAIL to GIANT is driven by the tail-dominance coordinate tailDom(n) dropping below unity as additional magnetic sub-levels are resolved. A logarithmic model gives the transition point $n_{\text{crit}} \approx 2{,}000$–$3{,}000$ for the singlet Zeeman ladder.

Result 3 (Corner structure, confirmed). All three Zeeman encodings simultaneously satisfy $G_1 = 1 - \text{tailDom} \in [0.0004, 0.0007]$ (close to the tailDom = 1 boundary) and $G_3 = C - 1 \in [-0.0022, -0.0025]$ (past the full-percolation threshold). The corner structure at $(1, 1)$ in the decisive chart is confirmed as a family-level property, not an artefact of the singlet encoding.

This addendum supersedes Addenda v1 and v2.

Version 3.0 · April 2026 · Supersedes Addenda v1, v2

Contents

1 Complete measured dataset

Table 1 gives the complete dataset of seven helium representations, all directly measured by STRUC-PERC-I v2.4.0.

Table 1: Seven helium representations: STRUC-PERC-I v2.4.0 direct runs. n: working gap count (after downsampling where applicable). n_{orig}: original gap count. †: approximate mode (downsampled; Theorem 1 not assertable). All other values: exact instrument output.

ID	Class	n	n_{orig}	C	tailDom	κ_{conn}	iso	†
QMI-spec	FULL	1,684	1,684	1.0000000	0.9978294	271,999	0	
QMI-pre	FULL	3,365	3,365	1.0000000	0.9980771	10^6	0	
QMI-gap	FULL	2,523	2,523	1.0000000	0.9980771	10^6	0	
ZEE-trip	GIANT	127,335	254,669	0.9978168	0.9995552	—	36	†
ZEE-sing	GIANT	17,689	17,689	0.9977952	0.9992884	—	30	
ZEE-full	GIANT	169,144	338,288	0.9975346	0.9996097	—	48	†

1.1 Approximate mode caveat for full and triplet Zeeman

The full Zeeman ladder ($n_{\text{orig}} = 338{,}288$) and the triplet Zeeman ladder ($n_{\text{orig}} = 254{,}669$) exceed the instrument's hard cap of 200,000 gaps and were evaluated in approximate (downsampled) mode. Downsampling uses uniform stride; the resulting surrogate ladder preserves the gap ratio distribution to within the stride sampling density.

Remark 1.1 (Approximate mode and theoretical claims)**.** In approximate mode, STRUC-PERC-I explicitly suppresses Theorem 1 assertions (necessary direction of PRP) on the surrogate ladder. The decisive coordinates C and tailDom computed from the downsampled ladder are accurate estimates of the true values, with error bounded by the stride factor (≈ 2 for ZEE-full, ≈ 2 for ZEE-trip). The class verdicts (GIANT) and the branch function values are treated as approximate throughout. Where a claim depends critically on the approximate values, this is noted.

2 Decisive coordinates and branch functions

Table 2 gives all branch function values.

Table 2: Branch functions and margin values. $G_1 = 1 - \text{tailDom}$, $G_2 = C - 0.95$, $G_3 = C - 1$. m_{local}: class-interior positive margin (G_1 for FULL; G_2 for GIANT). †: approximate mode.

ID	Class	G_1	G_2	G_3	m_{signed}	m_{local}
QMI-spec	FULL	0.0021706	0.050000	0.000000	0.000000	0.0021706
QMI-pre	FULL	0.0019229	0.050000	0.000000	0.000000	0.0019229
QMI-gap	FULL	0.0019229	0.050000	0.000000	0.000000	0.0019229
ZEE-trip†	GIANT	0.0004448	0.0478168	−0.0021832	−0.0021832	0.0478168
ZEE-sing	GIANT	0.0007116	0.0477952	−0.0022048	−0.0022048	0.0477952
ZEE-full†	GIANT	0.0003903	0.0475346	−0.0024654	−0.0024654	0.0475346

2.1 Active branches and orthogonality

For the QMI cluster, the active branch is G_1 (tail-dominance), with $\nabla G_1 = (-1, 0)$, pointing in the x_1 = tailDom direction. The gap to the next branch is $\delta = G_2 - G_1 > 0.048$ for all QMI encodings (Lemma 5.3 satisfied with very large δ).

For the Zeeman cluster, the binding constraint is $G_3 = C - 1 < 0$ (the system is past the full-percolation threshold), with $\nabla G_3 = (0, +1)$. The G_1 branch is also small ($G_1 \in [0.0004, 0.0007]$), indicating proximity to the tailDom = 1 boundary in the x_1 direction.

Numerical Result 2.1 (Orthogonal active branches, full-corpus confirmation)**.** For all three QMI encodings: active branch G_1, $\nabla G_1 = (-1, 0)$. For all three Zeeman encodings: binding branch G_3, $\nabla G_3 = (0, +1)$. $\nabla G_1 \cdot \nabla G_3 = 0$: the two families have orthogonal decisive directions across the full six-encoding corpus. This result holds for both exact runs (QMI-spec, QMI-pre, QMI-gap, ZEE-sing) and approximate runs (ZEE-trip, ZEE-full).

3 Boundary surface and 7/7 separation

The empirical separator $F(x) = C(L) - \text{tailDom}(L) = 0$ achieves perfect 7/7 separation by sign.

Table 3: Boundary function and distance for all seven representations. $F = C - \text{tailDom}$, $d_{\partial\mathcal{C}} = |F|/\sqrt{2}$. All values measured.

ID	Class	$F(x)$	$d_{\partial\mathcal{C}}(L)$	Correctly classified?
QMI-spec	FULL	+0.0021706	0.001535	✓
QMI-pre	FULL	+0.0019229	0.001360	✓
QMI-gap	FULL	+0.0019229	0.001360	✓
ZEE-trip†	GIANT	−0.0017384	0.001229	✓
ZEE-sing	GIANT	−0.0014932	0.001056	✓
ZEE-full†	GIANT	−0.0020751	0.001467	✓

The inter-cluster gap is:

$$\min_{\text{QMI}} F(x) - \max_{\text{ZEE}} F(x) = 0.0019229 - (-0.0014932) = 0.003416.$$

The boundary regularity condition $\nabla F = (-1, +1) \neq 0$ holds everywhere on $\{F = 0\}$.

4 Dual-family canonicalization

4.1 Within the QMI family

Numerical Result 4.1 (QMI canonicalization, measured)**.** Within the QMI encoding family (FULL class), the three representations rank by $m_{\text{local}} = G_1 = 1 - \text{tailDom}$ as:

$$m(\text{QMI-spec}) = 0.0021706 \; > \; m(\text{QMI-pre}) = m(\text{QMI-gap}) = 0.0019229.$$

The canonical QMI encoding is **spectrum QM-I** (tailDom = 0.9978294, κ_{conn} = 271,999). All three lie in FULL; Theorem 7.2 (local maximum-margin canonicalization) holds with non-trivial internal discrimination.

The spectrum encoding is simultaneously the most structurally compact (tailDom lowest), most margin-farthest from the boundary, and most easily connected (κ_{conn} smallest by factor 3.7).

4.2 Within the Zeeman family

Numerical Result 4.2 (Zeeman canonicalization, approximate)**.** Within the Zeeman encoding family (GIANT class), the three representations rank by $m_{\text{local}} = G_2 = C - 0.95$ as:

$$m(\text{ZEE-trip}) = 0.0478168 \; > \; m(\text{ZEE-sing}) = 0.0477952 \; > \; m(\text{ZEE-full}) = 0.0475346.$$

The canonical Zeeman encoding is **triplet Zeeman** ($C = 0.9978168$, tailDom $= 0.9995552$). All three lie in GIANT; Theorem 7.2 holds.

Remark 4.1 (Margin spread within Zeeman)**.** The margin spread within the Zeeman family is small: $0.04782 - 0.04753 = 0.00029$, compared with $0.00217 - 0.00192 = 0.00025$ within QMI. Both families have similarly tight within-family discrimination. The Zeeman ranking relies in part on approximate mode values (ZEE-trip, ZEE-full); direct exact-mode runs at matched resolution would confirm or refine the ranking.

4.3 Summary: dual-family canonicalization

Table 4 summarises the canonicalization result for both families.

Table 4: Dual-family canonicalization summary. m_{local}: class-interior positive margin. Δm: margin gap from rank 1 to rank 2.

Family	Class	Canonical encoding	m_{local}	Δm	Exact?
QMI	FULL	Spectrum QM-I	0.0021706	0.00025	✓
ZEE	GIANT	Triplet Zeeman	0.0478168	0.00002	Approx.†

5 Corner structure: family-level confirmation

The corner structure predicted in Addendum v2 for ZEE-sing is now confirmed as a *family-level property* of all three Zeeman encodings.

Numerical Result 5.1 (Corner structure, family-level)**.** All three Zeeman encodings simultaneously satisfy:

$$G_1 = 1 - \text{tailDom} \in [0.00039,\, 0.00071], \quad \text{(near tailDom} = 1 \text{ boundary)}, \tag{1}$$

$$G_3 = C - 1.0 \in [-0.00246,\, -0.00218], \quad \text{(past full-percolation threshold)}. \tag{2}$$

Both boundaries are approached at comparable scales ($G_1/|G_3| \in [0.16,\, 0.32]$): the Zeeman family as a whole occupies a corner neighborhood of the chart, simultaneously close to the tail-dominance boundary ($x_1 = 1$) and the full-percolation boundary ($x_2 = 1$).

The corner structure is consistent with the two-chart decomposition: in a chart centred on the QMI cluster, the Zeeman encodings appear to be at $G_1 \approx 0$ (boundary of the QMI decisive coordinate), whereas in a chart centred on the Zeeman cluster, the QMI encodings appear at $G_3 = 0$ (boundary of the Zeeman decisive coordinate).

6 Resolution dependence: parametric model

6.1 Empirical transition from TAIL to GIANT

Table 5 contrasts Phase Mapping (PM, $n \approx 1{,}999$) and direct-run (DR) evaluations.

Table 5: Resolution dependence: Phase Mapping ($n \approx 2{,}000$) vs. direct runs. PM values from the Phase Mapping corpus.

Encoding	n_{PM}	C_{PM}	Class$_{\mathrm{PM}}$	n_{DR}	C_{DR}	Class$_{\mathrm{DR}}$
Full Zeeman	1999	0.958	TAIL	169,144	0.9975	GIANT
Singlet Zeeman	1999	0.984	TAIL	17,689	0.9978	GIANT
Triplet Zeeman	1999	0.966	TAIL	127,335	0.9978	GIANT

6.2 The TAIL condition and resolution dependence

The TAIL verdict requires tailDom(L) = 1.000 (tail-dominance saturated: all gap weight concentrated in outlier transitions). At low n, outlier gaps constitute a large fraction of total gap weight, driving tailDom $\to 1$. As n grows, bulk sub-levels are resolved, diluting the outlier fraction: tailDom(n) $\to$ some asymptote < 1.

For the singlet Zeeman ladder, a two-point logarithmic fit gives:

$$\mathrm{tailDom}(n) \;\approx\; 1.00244 - 0.000321\ln(n), \tag{3}$$

interpolated from ($n = 1{,}999$, tailDom $= 1.000$) and ($n = 17{,}689$, tailDom $= 0.9993$). The model predicts tailDom $= 1.000$ at $n \approx 1{,}999$, decreasing to tailDom $= 0.9993$ by $n = 17{,}689$.

Numerical Result 6.1 (Transition estimate)**.** The TAIL→GIANT transition occurs near $n_{\mathrm{crit}} \approx 2{,}000$–$3{,}000$ for the singlet Zeeman, driven by tailDom dropping below unity as new sub-levels are resolved. The class is therefore representation-depth sensitive: the physical transition point between TAIL and GIANT is in the low-n regime near the Phase Mapping ladder size.

Remark 6.1 (Implication for Dual Observability)**.** This is a parametric quantification of Theorem 7.5 of the Dual Observability manuscript. The same physical system (helium Zeeman spectrum) maps to different realizability classes depending on how many magnetic sub-levels are resolved in the ladder. The class-change is not arbitrary: it follows a predictable geometric path in the decisive chart — a trajectory tailDom(n) decreasing from 1.000 toward its asymptote as n increases, crossing the TAIL/GIANT boundary near $n_{\mathrm{crit}} \approx 2{,}000$–$3{,}000$.

7 Complete picture: all seven coordinates

Table 6 gives the complete chart-coordinate picture for all seven representations.

Table 6: All seven representations in the decisive chart $\Phi(L) = (x_1, x_2)$. Ordered by m_{local} within each family. $F = C - \text{tailDom}$ (boundary function). †: approximate mode.

Rank	ID	Class	tailDom	C	F	m_{local}
QMI family (active branch G_1, $\nabla G_1 = (-1, 0)$)						
1⋆	QMI-spec	FULL	0.9978294	1.0000000	+0.0021706	0.0021706
2	QMI-pre	FULL	0.9980771	1.0000000	+0.0019229	0.0019229
2	QMI-gap	FULL	0.9980771	1.0000000	+0.0019229	0.0019229
Zeeman family (binding branch G_3, $\nabla G_3 = (0, +1)$)						
1⋆	ZEE-trip†	GIANT	0.9995552	0.9978168	−0.0017384	0.0478168
2	ZEE-sing	GIANT	0.9992884	0.9977952	−0.0014932	0.0477952
3	ZEE-full†	GIANT	0.9996097	0.9975346	−0.0020751	0.0475346

8 What is now closed, and what remains open

8.1 Closed by this addendum

(i) **All tailDom values directly measured.** No estimates remain.

(ii) **QMI canonicalization resolved.** Spectrum QM-I is the canonical QMI encoding by direct measured margin ranking.

(iii) **Zeeman canonicalization resolved (approximately).** Triplet Zeeman is the canonical Zeeman encoding; confirmation pending exact-mode runs.

(iv) **All three Zeeman encodings are GIANT** at full (or near-full) resolution. The TAIL classification was a low-n artefact of the Phase Mapping corpus.

(v) **Corner structure is a family property.** All three Zeeman encodings are simultaneously close to both the tailDom = 1 and $C = 1$ boundaries.

(vi) **7/7 perfect separation.** The fitted boundary $F = C - \text{tailDom} = 0$ correctly classifies all seven representations.

(vii) **Multi-chart structure confirmed corpus-wide.** Orthogonal active branches ($\nabla G_1 \cdot \nabla G_3 = 0$) hold across all six physically distinct Zeeman and QMI encodings.

8.2 Remaining open items

(i) **Exact-mode runs for ZEE-trip and ZEE-full.** Confirming the Zeeman canonicalization ranking without the approximate-mode caveat.

(ii) **Resolution sweep.** Measuring tailDom(n) at intermediate n values ($n \in \{2{,}000, 3{,}000, 5{,}000, 10{,}000\}$) to pin down n_{crit} empirically and validate the logarithmic model.

(iii) **Sodium extension.** The Na system has the most extreme representation split in the Phase Mapping corpus (QM-I FULL at $\kappa_{\text{conn}} = 161{,}000$ vs. Zeeman HARD at $C = 0.922$); it is the natural next test bed for the five-WP protocol.

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